Low-Speed Stability - Erata

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In a March 2004 article titled *Low-Speed Stability* , I discussed the three major moment forces affecting the longitudinal balance of an aircraft. The primary motivation for the previous article was to look at the longitudinal forces based on the C_m of the airfoil instead of the center of pressure. It also showed how airspeed becomes increasingly more sensitive to elevator trim setting as the CG is moved rearward.

Unfortunately, I incorrectly stated that "*An aircraft can be stable at higher speeds but unstable at low speeds*." While this statement may not be absolutely false, it certainly isn't true simply because the tail is producing a lifting force, as suggested in the article, nor was it the motivation for the article. I misinterpreted statements made by Simons[1] about lifting tail designs. What has become clearer after reading Etkin [2], is that like an airfoil, the entire aircraft has a moment coefficient, and how it changes with changes in pitch determines longitudinal stability.

$$C_m = C_{mo_w} + C_{L_w} (h - h_{n_w}) - V_H C_{L_t}$$

where C_{mo_w} is the moment coefficient of the wing, C_{L_w} is the lift coefficient of the wing, h and h_{n_w} are locations, as a percentage of the mean wing chord, respectively of the CG and the aerodynamic center (neutral point) of the wing, V_H is the tail volume coefficient, and C_{L_t} is the lift coefficient of the tail.

Since this equation is in terms of aerodynamic coefficients, it is independent of airspeed. However, it does depend on the angle of attack, a, which affects both the wing and tail lift coefficients, and it is the angle of attack, or pitch, that we wish to be stable. The following equation expresses the total aircraft moment in terms of a

$$C_m = C_{mo_w} + a\ [a_w (h - h_{n_w}) - a_t V_H (1 - (\P e /\P a)\ ] + V_H a_t (e_0 + i_t)$$

where a_w is the lift slope of the wing, a_t is the lift slope of the tail, i_t is the tail incidence angle, e_0 is the downwash angle at the zero lift a, and $\partial e / \partial a$ represents the the change in downwash angle as a changes[3]. In this equation, the tail angles, i_t and e_0, are positive when the leading edge is lower than the trailing edge.

This equation shows that the tail moment has two components one that varies with a and one that depends on the elevator trim. The value of a affects the lift coefficients of both the wing and tail. The sum of the lift terms must equal the sum of the airfoil moment, C_{mo_w}, and elevator trim.

For a specific elevator trim setting, there is one a where C_m is zero, where the aircraft is *balanced*. Changing the elevator trim changes the a that zeroes the aircraft moment. This a also determines the airspeed where the lift terms equal the weight of the aircraft. The following equation determines how much a change in a changes C_m above

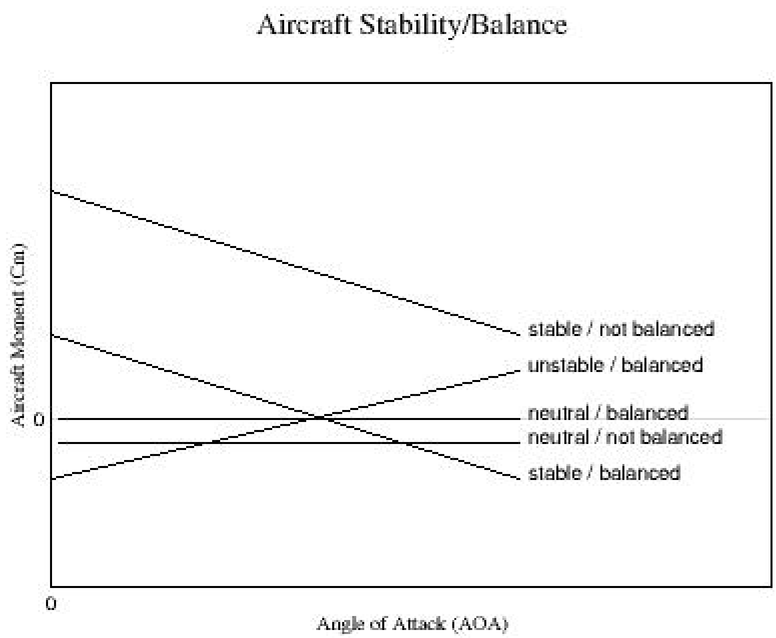
$$C_{ma} = \P C_m /\P a = a_w [(h - h_{n_w}) - V_H (a_t / a_w) (1 - (\P e /\P a)]$$

For a specific aircraft design, all the variables in this equation are constant. However, the CG location, h, can more easily be changed. If C_{ma} is a positive value, an increase in a makes the moment more positive, pushing the nose upward. If C_{ma} is negative, an increase in a decreases the moment force, pushing the nose downward. C_{ma} may also be zero, meaning a change in a has no affect on the moment force. A stable aircraft has a negative C_{ma}, when tubulence causes a to increase, C_m becomes more negative to force the nose down.

The value of h can make C_{ma} positive, negative or zero. Increasing h, moving the CG rearward, increases the value of C_{ma}. At some point the value of h makes C_{ma} zero. This is the *neutral point* of the aircraft, h_n, the CG location where a change in a has no affect on C_m. Locating h behind h_n will make C_{ma} positive, and locating h in front of h_n will make C_{ma} negative. The above equation can be rewritten to determine that value.

$$h_n = h_{n_w} + V_H (a_t / a_w) (1 - ¶ e /¶ a)$$

Static *stability* requires that C_{ma} be negative, that the CG be in front of the neutral point, $h < h_n$. For an aircraft to be *balanced*, the value of C_m must be zero at some reasonable value of a. However, while an aircraft must be balanced, pilot preference determines the desired level of stability.



The figure illustrates various possibilities of balance and stability. Changing the elevator trim shifts the curve up or down. Both the stable and neutral but not-balanced cases simply require elevator trim adjustment to make them balanced.

At this point it should be clear that there is no one term in the equations above, such as a lifting tail, that cause instability. It is the aerodynamic design, the combination of elements, that determine stability, and stability is only one aspect of a design.

On a tailless aircraft, the equation for C_m contains no elevator terms.

$$C_m = C_{mo_w} + C_L (h - h_{n_w})$$

The aircraft neutral point is the same the wing's, $h_n = h_{n_w}$. Again, h_n determines the CG location where a change in a has no affect on C_m, and this determine stability. In this case, if turbulence pushes the nose up, a increases, increasing C_L, and since the aerodynamic center of the wing is behind the CG, the increased lift pushes the nose back down. Stability requires that the CG be in front of the neutral point, and balance requires that C_m be zero.

To replace the elevator function, some mechanism is needed that affects C_{mo} and/or C_L. On tailless aircraft, rather than use the tail to counter balance the airfoil and lift moments, part or all of the wing's trailing edge is used to change the airfoil shape, affecting C_{mo} and C_L directly. The following equation describes the change of C_m in terms of the trailing edge deflection, d,

$$C_{md} = \P\, C_{mo} /\P\, d + C_{Ld}\, (h - h_n)$$

where $\P\, C_{mo} /\P\, d$ is the change in the airfoil moment coefficient, and C_{Ld} is the change in the lift coefficient, as the trailing edge is deflected.

If a downward adjustment of d is considered positive, $\P\, C_{mo} /\P\, d$ is typically negative and C_{Ld} is positive. Adjusting d downward makes the airfoil moment more negative, and shifts the lift curve upward, more positive. Inversely, adjusting d upwards and makes the airfoil moment more positive, and lift coefficient more negative [4]. While the total affect changes in d has on C_m is relatively small, the longitudinal moment of inertia of a tailless aircraft is smaller than a conventional aircraft, and less moment force is required to affect pitch control.

While these changes affect C_{mo} and C_L in opposite directions, a positive change in C_L causes a negative change in C_m because the CG is in front of the neutral point and $h - h_n$ is negative. The negative of this difference, $h_n - h$, is the *static margin*. It is also the moment arm that determines how much the lift affects C_m.

Since C_L is typically always positive, it will always contribute to a negative C_m depending on the amount of static margin. This increases the need for C_{mo} be positive. While increasing the static margin increases stability, it also negatively affects the trailing edge effectiveness in controlling pitch.

On an unswept constant chord wing, the aerodynamic center at each span position is at the same longitudinal location. For a swept wing, the aerodynamic center of the entire wing, the neutral point, is the aerodynamic/geometric mean of the aerodynamic center across the entire span. Partial trailing edge adjustments, affecting the lift at different span positions, will shift the neutral point of the wing.

If the lift is increased on the more rearward outer portions of the wing, the aerodynamic center of the lift generated by the entire wing shifts closer (rearward) to the aerodynamic centers of those portions of the wing generating the increased lift. Conversely, if the outer portions of the wing produces less lift, the neutral point shifts forward.

When the neutral point moves rearward, away from the CG, the static margin and moment arm of the lift increase. This not only affects stability, but also how much the lift affects C_m. Increasing the lift on the rearward portions will make C_m more negative, causing a nose-down pitch change.

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1. *Model Aircraft Aerodynamics 4th ed.*, Simons, Martin; Nexus Special Interests, 1999, pg 243.

2. *Dynamics of Flight: Stability and Control 2nd ed.*, Etkin, Bernard; John wiley and Sons, New York, 1982,

3. Simons suggests that $\partial e / \partial a$ can be approximated by the value $35a_w/A$, where A is the aspect ratio

of the wing. Simons also indicates that the tail efficiency needs to be considered and suggests scaling the tail volume by 0.65 for a normal tail and 0.9 for a T-tail.

4. For the RG15 from the UIUC database, which has measurements for the RG15 at different flap settings, ¶ C_{mo} /¶ d is -0.0033, and C_{Ld} is 0.0215.